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ABSTRACT

In response to citizens' need and right to know about how North Dakota students perform in mathematics, the Department of Public Instruction and the North Dakota Council of Teachers of Mathematics administered a criterion-referenced mathematics assessment in grades 4 (1,298 students) and 8 (1,475 students). Also included were items from the National Assessment of Educational Progress. The objectives were classified into four process categories: Mathematical Knowledge, Mathematical Skill, Mathematical Understanding, and Mathematical Application and Problem Solving. Only cognitive skills were assessed, without consideration of variables such as per pupil expenditure, ethnicity, and family income. Different tests were administered at the fourth and eighth grades, the intent being to measure performance at both a basic level and at an appropriate level of difficulty for both grades. Twenty-five of the items appearing on the fourth grade level, however, were also included at the eighth grade level. When compared with national results, children at both grade levels performed very well in assessment exercises. Some areas (place value, geometric terminology, metric system, graph interpretation, word problems), however were found in need of attention. (Author/GSK)

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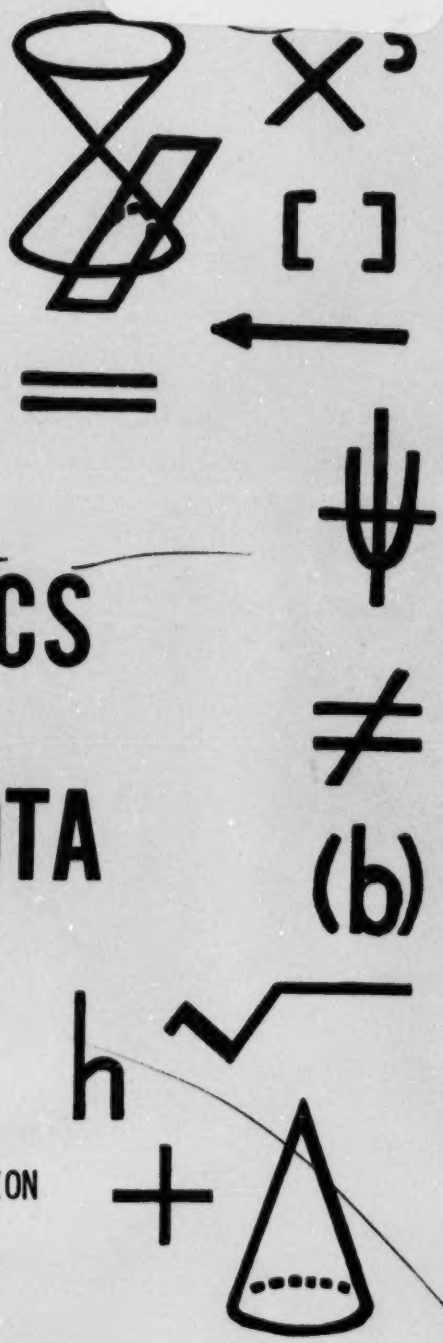
SUMMARY OF PERFORMANCE FOR
FOURTH AND EIGHTH GRADE
MATHEMATICS ASSESSMENT
SPRING, 1979



MATHEMATICS IN NORTH DAKOTA

NORTH DAKOTA DEPARTMENT OF PUBLIC INSTRUCTION
H. J. SNORTLAND, SUPERINTENDENT
BISMARCK, NORTH DAKOTA
OCTOBER, 1979

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SUMMARY OF PERFORMANCE FOR
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OCTOBER, 1979

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ACKNOWLEDGMENTS

This is the report of the North Dakota Mathematics Assessment for 1978-79. It includes an analysis of the results of the mathematics tests administered in April, 1979, to grade 4 and 8 students enrolled in randomly selected schools.

The assessment could not have been accomplished without the cooperation of educators throughout the state. Therefore, we wish to extend our sincerest appreciation to the administrators and teachers who coordinated the administration of the tests in the schools and to the following individuals who served on the Advisory Committee.

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CHAPTER I

Mathematics is generally recognized as one of the most important basic skills necessary for coping with life in an increasingly complex society.

In trying to respond to citizens' need and right to know about how North Dakota students are performing in this basic skill, the Department of Public Instruction with the cooperation of the North Dakota Council of Teachers of Mathematics administered a criterion-referenced mathematics assessment in grades 4 and 8.

Assessment programs that focus on measuring educational performance with respect to very specific educational objectives through the use of test items have been termed criterion-referenced; this means that the interpretation of results is based on the test content rather than a comparison to a national or local norm group.

This approach to assessment tends to measure educational performance across a rather narrow band of possible educational outcomes. No attempt was made in this assessment to go beyond the academic area by trying to measure student outcomes related to affective influences.

While the criterion-referenced approach to assessment was used, standardized norm-referenced tests have a legitimate place in educational measurement because they provide information about comparative levels of achievement which educators and the general public want and need. To some extent comparisons in this assessment have been accomplished by including items made available by the National Assessment of Educational Progress

(NAEP). The inclusion of NAEP exercises enabled a comparison to be made of North Dakota student performance with the performance of national groups as measured by the NAEP program. The students who participated in the 1979 North Dakota Statewide Assessment project were in the fourth and eighth grades. The performance of grade 11 students was assessed during the 1978 school year and the results were distributed at the beginning of the 1978-79 school year.

The assessment instruments were administered during April, 1979, by the staff of each school selected to participate in the assessment study. While an approximate time for test completion was suggested, it was considered liberal enough for all students to answer the test items.

The basic sampling approach which was employed involved the selection of a stratified random sample of schools. Enrollment was used as a basis for stratification with a maximum of 30 students from any one school. In schools where the sample did not exhaust the number of students enrolled in the designated grades, the participants were selected in the following manner: Using the designated grade enrollment list, students were selected in the order in which the first letter of their last name appeared in a list of randomized letters. Beginning with the top row, schools were instructed to read the letters from left to right, then to proceed to the second row, etc. Schools were instructed to cycle through the randomized letters as often as necessary, but no more than one student was to be selected for each letter during each cycle.

As with National Assessment, the North Dakota Assessment does not use total performance scores for individual respondents because its main concern is how the two groups of individuals perform on specific exercises. Thus, it was not necessary for each respondent to take every exercise. The exercises were sorted into four forms of the test, and each in-school

respondent took only one form. This technique makes it possible to assess performance on far more exercises than would be possible in the usual testing situation and provides broader coverage of the objectives in mathematics.

Not every district in the state was included in the sample, therefore, the results of the project cannot be used to compare the students from one district to another. The results may be used to indicate the general level of effectiveness of the educational system within the State of North Dakota on the mathematics objectives assessed.

At the fourth grade 1,298 students out of an enrollment of 9,660 were tested. At the eighth grade 1,475 students out of an enrollment of 11,191 were tested.

The criterion-referenced tests were assembled by the North Dakota Department of Public Instruction with the advice of a committee consisting of local school teachers and representatives from the colleges in the state.

Objectives to be measured by the criterion-referenced tests were chosen on the basis of their significance to basic mathematics skills. They are not proffered as a comprehensive coverage of the entire domain of mathematics but are an attempt to answer some basic questions concerning the cognitive skills of North Dakota students in mathematics. In grade 4 a total of 52 objectives were assessed and 59 at the eighth grade level. The answers to these questions also provide information needed to answer criticism about the adequacy of education in American schools.

The assessment specifically tries to answer questions such as:

-Can students add whole numbers?
-Can students add decimals?
-Can students subtract whole numbers?
-Can students understand the metric system?
-Can students work consumer problems?

These and many other basic skills are assessed through items specifically selected for this purpose.

It should be noted that only cognitive skills were assessed without consideration of variables such as per pupil expenditure, ethnicity and family income. While it is recognized that performance is related to selected school and home factors which are believed to influence student achievement in mathematics, these factors in North Dakota do not vary significantly, and therefore, their impact was not assessed.

CHAPTER II

OBJECTIVES

The objectives presented here provide the basis for the selection of exercises to assess the achievement, in mathematics, of fourth and eighth grade students in North Dakota. The design chosen for the North Dakota Statewide Mathematics Assessment by the Advisory Committee is similar to the one used by the National Assessment of Educational Progress (NAEP). NAEP is an annual national survey designed to provide information about educational attainments of children and young adults. The objectives are adapted from those developed for the Minnesota Educational Assessment program.

The objectives were classified into four process categories. Although each category suggests a type of mental process, neither objectives nor exercises fall neatly into a single process category--if only because the process has to be inferred; and different people use different processes or different combinations of processes. Arbitrary decisions must be made in using any system of process categories. Such a system is helpful, however, in ensuring consideration of the diversity possible within a given content category.

The objectives for which test items were included in the assessment are denoted with an asterisk (*).

MATHEMATICS OBJECTIVES
GRADE 4

1. MATHEMATICAL KNOWLEDGE

Mathematical knowledge refers to the recall and recognition of mathematical ideas expressed in words, symbols or figures. Mathematical knowledge relies, for the most part, on memory processes. It does not ordinarily require other, more complex mental processes.

Exercises that assess mathematical knowledge require that a person recall or recognize one or more items of information. An example of an exercise involving recall would be one that asks for a multiplication fact such as the product of five and two. Another example would be an exercise asking for the statement of a mathematical relationship such as the law of cosines. An example of an exercise involving recognition would be one that presents several symbols and asks which symbol means "parallel."

A. Facts of Arithmetic

The student will:

*1. recall the basic facts of addition with addends less than 11, in both horizontal and vertical formats

*2. recall the basic facts of subtraction, $c - b = a$, where $a, b < 11$, in both horizontal and vertical formats

*3. recall the basic facts of multiplication, $a \times b = c$, where $a < 6$ and $b < 10$

*4. recall the basic facts of division, $c \div a = b$, where $a < 6$ and $c < 46$

B. Numeration

The student will:

*1. identify by written word, the counting numbers to 100

*2. identify by numerical symbol, the counting numbers to 1,000

3. read and write Roman numerals to XII

4. identify by written word and numerical symbol, the unit fractions with denominators of 2, 3, 4, 5 and 6

*5. recognize examples of the following: whole numbers, equality, addition, subtraction, multiplication, division, sum, difference, product, place value, odd, even, units of measure, and one, two,...nine

6. recognize examples of the following terms: divisor, quotient, remainder, addend

*7. when given a drawing of a number of objects fewer than 10, will correctly identify the order or position of objects

C. Geometry

The student will:

*1. recognize examples of square, rectangle, triangle, circle, right angle, intersecting lines, parallel lines

*2. identify the following by matching the name with an appropriate pictorial representation of the concept: point, line, curve (open and closed), cube

3. when given a closed curve and a point, identify whether the point is inside, on or outside the curve

D. Measurement

The student will:

1. given a metre stick, identify a metre and centimetre

2. given a yardstick, identify an inch, a foot and a yard

*3. given pictures of coins, identify a penny, nickel, dime and quarter and tell its value in cents

*4. identify the quantitative relationship among: centimetre and metre; inch, foot and yard; gram and kilogram; ounce and pound; second, minute, hour, day, month and year; and units of U.S. currency

*5. convert units of measurement within a measurement system

E. Algebra

*1. The student will recognize proper use of the symbols: ( ), =, ≠, < and >

F. Sets

The student will:

1. recognize examples of equivalent and non-equivalent sets
2. when given two sets, identify elements which are in both sets
3. identify examples of the empty set

2. MATHEMATICAL SKILL

Mathematical skill refers to the routine manipulation of mathematical ideas. Mathematical skill relies on algorithmic processes. An algorithm is a standard procedure that always leads to an answer. Mathematical skill requires the recollection of how to use the algorithm.

Exercises that assess mathematical skill assume that the required algorithm has been learned and practiced. They do not require that the person decide which algorithm to use or that he or she apply the algorithm to a new situation. Such an exercise aims at measuring proficiency in carrying out the algorithm rather than the understanding of how or why it works. Mathematical skill is assessed by exercises that require the performance of specified tasks such as making measurements, multiplying two fractions, graphing a linear equation or bisecting an angle.

A. Computation and Numeration

The student will demonstrate the following abilities without the use of a calculator:

1. Addition (both horizontal and vertical formats)

   *a. two whole numbers, each less than 1,000, with and without regrouping

   *b. three whole numbers, each less than 1,000, with and without regrouping

   *c. decimals

*2. Subtraction (both horizontal and vertical formats)

find the difference between two numbers less than 1,000 with and without regrouping

*3. Multiplication

name the product of any two digit number and any one digit number less than 6 with and without regrouping

*4. Division

when given a dividend less than 50 and a one digit divisor, name the quotient and remainder

*5. when given any number less than 1,000, correctly round it off to the nearest hundred and the nearest ten

6. when given a table of addition or multiplication of two sets of whole numbers, find the sum or product of any two numbers in the sets

7. when given a rectangular array of numbers, identify the element in the n-th row and the k-th column

8. list ten consecutive even or odd numbers given and counting number less than 1,000 to start with

*9. count by 2's, 3's, 5's or 10's, starting with 2, 3, 5 or 10 respectively, for at least ten numbers

B. Algebra

The student will:

1. given a blank number line, correctly place the numbers 0-10 on it
2. given a simple equation (one variable, one operation), graph its solution on a number line
3. given a simple equation, select its solution from a specified domain, i.e., {1, 2, 3, 4, 5}; which one solves n + 6 = 10

C. Measurement

The student will:

*1. when given a ruler marked off in arbitrary units, measure a given segment to the nearest unit
2. when given a picture of a clock face without hands, draw the hands to indicate a given time to the hour, half-hour, quarter-hour and five minutes
*3. when given a clock face with a time indicated, will write the time in the form 6:45 to the nearest 5 minutes
*4. correctly read the temperature on an indicated thermometer
*5. when given groups of pennies, nickels, dimes and quarters (some of each) totaling less than $1.00, name the total value in cents
6. when given a written amount of money less than $1.00, select coins totaling that amount in value
7. when given common daily occurrences, label the times of those occurrences as a.m. or p.m.
8. for a purchase less than $1.00, select the set of coins, from four given sets, which would make the correct change

D. Probability and Statistics

The student will:

*1. when asked questions about a given graph, name the correct

data to answer the question

2. tell what one might most likely draw when choosing from a set of different numbers (without looking) of 2 or more kinds of objects, e.g., 7 red and 3 green balls
3. record the outcomes of probability events in the form "3 out of 5"
4. after observing a probability experiment and its outcome, give the most likely outcome in a repetition of the same experiment

3. MATHEMATICAL UNDERSTANDING

Mathematical understanding refers to the explanation and interpretation of mathematical knowledge. Mathematical understanding relies primarily on translation processes. The mathematical knowledge can be expressed in words, symbols or figures; and the translation may be within or between any of these modes of expression. Mathematical understanding involves memory processes as well as processes of associating one item of knowledge with another.

Exercises that assess mathematical understanding require that a person provide an explanation or an illustration for one or more items of knowledge. They require the transformation of knowledge but not the application of that knowledge to the solution of a problem. An example of an exercise involving explanation would be one that asks why a certain graph is not the graph of a function. Examples of exercises involving interpretation would be ones that ask for a drawing of an array to represent six times seven or that ask for an equation to represent the information in a word problem.

A. Algebraic Expressions

The student will:

1. explain the meaning of number frames such as $\square$ and $\triangle$
*2. when given a simple word problem, select from a number of alternatives the number sentence that correctly fits the problem
3. demonstrate the commutative property of addition by correctly indicating that the commuted form of a statement is

equivalent to the original, i.e., 5 + 7 = 7 + 5

4. recognize applications of the associative property of addition by indicating that two sums, arrived at by regrouping the addends, are equal

*5. give the sum of any number and zero as the given number

6. demonstrate the commutative property of multiplication by correctly indicating that the commuted form of the factors give the same result as the original form

7. recognize applications of the associative property of multiplication by indicating the equality of products of three one-digit factors, associated in two ways

*8. give the product of any number and 1 as the given number

*9. given any two counting numbers less than 1,000 in a certain order, write <, = or >, whichever is the correct symbol, between the two

B. Graphing

The student will:

1. correctly select from three or more choices, the coordinates of a point with integral coordinates in the first quadrant

2. when given the integral coordinates of a point in the first quadrant, correctly plot the point

C. Numeration

The student will:

*1. when given any numeral less than 1,000, assign the correct place value to each of the digits

*2. when given any numeral less than 1,000, rewrite the number in expanded notation (not exponential form)

*3. correctly match number sentences with set combination drawings (addition)

*4. correctly match number sentences with set and subset removal drawings (subtraction)

5. identify "how many fewer objects" in one set than in another

*6. indicate that repeated addition can be done by multiplication

7. indicate, without counting, the number of symbols in a rectangular array (less than 25 objects)

8. find the area of rectangles with integral sides less than 6 units with the unit squares drawn in

9. correlate a division statement with a drawing showing the partitions of a set

*10. find the fractional part of a number

11. when given a set of 30 or fewer objects, name the number of equivalent sets of cardinality 5 or less and the number "left over"

*12. when given a drawing of a whole divided into 5 or fewer equal parts, identify the fractional part which is shaded

13. when given a fraction with a denominator less than 9, correctly shade the corresponding parts of a given region in a drawing

14. correctly match number sentences with addition arrows on a number line

*15. correctly match number sentences with subtraction arrows on a number line

16. correctly identify different names for the same number, e.g., 3 + 5, 8, 12 - 4, etc.

17. when given a sequence of fewer than 10 symbols, name the ordinal position of any of the symbols

D. Measurement

The student will:

*1. when given examples of different objects whose length or distance is to be measured, select the appropriate metric unit to use

*2. when given various items to find the mass of (to be weighed), select the appropriate metric unit to use

*3. determine time elapsed

4. MATHEMATICAL APPLICATION AND PROBLEM SOLVING

Mathematical application refers to the use of mathematical knowledge, skill and understanding. Mathematical application relies on memory, algorithmic, translation and judgment processes.

Exercises that assess mathematical application require a sequence of processes that relate to the formulation, solution and interpretation of problems. The processes may include recalling and recording knowledge, selecting and carrying out algorithms, making and testing conjectures and evaluating arguments and results. An exercise might require the solution of a standard problem on proportion, or it might require the demonstration that two geometric figures are congruent. It might require the estimation of the amount of carpet needed for a room or the formulation of a problem given a graph of statistical data. Exercises assessing mathematical application may vary from routine textbook problems to exercises dealing with mathematical arguments. In these exercises the person is not told exactly what to do or how to do it. He or she must use reasoning and decision-making processes as well as mathematical knowledge, skill and understanding.

A. Recognizing General Patterns and Reasoning in Arithmetical Problems

The student will:

*1. when given a computation or a problem and asked to estimate the answer, give an estimation within one order of magnitude

*2. when given a set of five different counting numbers less than 1,000, arrange them from smallest to largest

*3. when given a simple word problem, find the solution

*4. determine the distance traveled from two odometer readings

*5. when given a problem with extraneous data, correctly identify that data as extraneous

*6. when given a problem with insufficient data, correctly identify data which is needed to solve the problem

*7. when given any set of unit fractions with denominators less than 6, arrange them from smallest to largest

B. Geometric Generalizations and Relations

The student will:

*1. find the perimeters of rectangles, triangles and other polygons

2. when given pairs or sets of geometric figures, identify similarities among the figures, such as number of sides, curved boundaries, etc.

3. when given pairs or sets of geometric figures, identify differences among the figures, such as size, shape, number of sides, etc.

C. Consumer Problems

The student will:

*1. when given items at different prices, the student will determine total cost

*2. make change from a dollar for a purchase less than a dollar

*3. make change from an amount less than a dollar

5. APPRECIATION AND USE OF MATHEMATICS

The student will:

1. recognize the importance and relevance of mathematics to the individual and to society
2. recognize that mathematics can be enjoyable

MATHEMATICS OBJECTIVES
GRADE 8

1. MATHEMATICAL KNOWLEDGE

Mathematical knowledge refers to the recall and recognition of mathematical ideas expressed in words, symbols or figures. Mathematical knowledge relies, for the most part, on memory processes. It does not ordinarily require other, more complex mental processes.

Exercises that assess mathematical knowledge require that a person recall or recognize one or more items of information. An example of an exercise involving recall would be one that asks for a multiplication fact such as the product of five and two. Another example would be an exercise asking for the statement of a mathematical relationship such as the law of cosines. An example of an exercise involving recognition would be one that presents several symbols and asks which symbol means "parallel."

*A. Facts of Arithmetic

The student will recall the basic facts of addition, subtraction, multiplication and division, where the addends or the factors are whole numbers 0 through 9.

B. Numeration

The student will do the following:

1. read and write numbers in base 10 (thousandths through million)

*2. recognize examples of the following terms: factor, multiple, decimal, fraction, prime number, square root, percent, ratio, integer

3. identify the meaning of the symbols: %, $\frac{a}{b}$, $\sqrt{\ }$, π

*4. identify the value of the digits of a number written in base 10 (place value)

5. recognize that fractions and decimals are two forms of a rational number

6. read and write numbers in scientific notation

*7. read and write Roman numerals

C. Geometry

The student will do the following:

*1. recognize examples of the following terms: point, line, plane, segment, angle, polygon, circle, right angle, parallel, perpendicular, perimeter, diameter, radius, circumference, triangle, square, rectangle, quadrilateral, parallelogram, hexagon, octagon, cube, base, altitude

2. recall that the value of pi is 3.14...

*3. given possible definitions of the following terms, choose the correct one for each: segment, polygon, circle, right angle, parallel, perpendicular, perimeter, diameter, radius, circumference, triangle, square, rectangle, quadrilateral, parallelogram, hexagon, octagon, cube

4. recognize the symbols: $\angle$, $\triangle$, $\perp$, $\parallel$

D. Measurement

The student will identify:

*1. kilometre, metre, centimetre and millimetre as units of length; kilogram and gram as units of mass (weight); litre and millilitre as units of capacity; and degrees Celsius as the unit of temperature

*2. the values associated with the metric prefixes: kilo, hecto, deka, deci, centi and milli

*3. the term area as meaning the amount of surface and the term volume as meaning the amount of space enclosed

*4. the quantitative relationships among units of measurement

E. Algebra

The student will recognize examples of and correctly use:

*1. symbols: =, exponent, >, <, ≠, ≥, ≤

2. terms: equation, variable (placeholder), inequality

F. Probability and Statistics

The student will:

1. use the following terms in the context of everyday activities: probability, chance of, odds, average

G. Technological Hardware

The student will identify examples of the following terms: data, flow chart, computer, calculator, input, output

2. MATHEMATICAL SKILL

Mathematical skill refers to the routine manipulation of mathematical ideas. Mathematical skill relies on algorithmic processes. An algorithm is a standard procedure that always leads to an answer. Mathematical skill requires the recollection of how to use the algorithm.

Exercises that assess mathematical skill assume that the required algorithm has been learned and practiced. They do not require that the person decide which algorithm to use or that he or she apply the algorithm to a new situation. Such an exercise aims at measuring proficiency in carrying out the algorithm rather than the understanding of how or why it works. Mathematical skill is assessed by exercises that require the performance of specified tasks such as making measurements, multiplying two fractions, graphing a linear equation or bisecting an angle.

A. Computation

The student will demonstrate the following abilities without the use of a calculator:

1. Addition and subtraction
   - *a. whole numbers
   - *b. decimals
   - *c. fractions and mixed numbers (with denominators less than 11)
   - *d. integers
2. Multiplication
   - *a. whole numbers (one of the factors with 2 or less digits)
   - *b. decimals (one of the factors with 3 or less digits)
   - *c. fractions and mixed numbers - denominators less than 11 (includes combinations)
   - *d. integers (1 digit less than 5)
3. Division
   - *a. whole numbers (3 or less digit divisor, 5 or less digit dividend, quotient to 2 decimal places)
   - *b. decimals (2 or less digit divisor, 3 or less digit dividend, quotient to 2 decimal places)
   - c. integers (problems involving basic facts)
4. given a fraction with the denominator a one- or two-digit number, change to a decimal rounded to nearest 3 decimal places

*5. change a decimal to an equivalent percent and vice versa

6. given a decimal with 2 or less decimal places, change to fraction in lowest terms

*7. arrange decimals and whole numbers according to numerical order

8. given two fractions with denominators less than 11, pick out the larger or smaller

*9. round off numerals to nearest ten, hundred, thousand or million

*10. round off decimal numerals to nearest tenth, hundredth or thousandth

11. convert repeating decimals to fractions

12. determine prime factorization of whole numbers with 3 or less digits

*13. determine G.C.F., L.C.M. of whole numbers with 2 or less digits

*14. solve three types of percentage problems: finding a percent of a number, finding what percent one number is of another and a number that is a given percent of another

15. rename fractions with equivalent fractions

16. find square roots in a table

17. find square roots by approximation, division or algorithm

B. Algebra

The student will:

1. use an equation, such as a = bc or a = b + c, to find the value of one variable given the values of two variables

2. recognize examples of the commutative, associative and distributive principles

*3. find rational number solutions for simple linear equations ("simple" shall mean one variable written only on one side of the equal sign)

4. determine the relationship of two ratios (>, <, =)

*5. given three values in a proportion, solve for the fourth

C. Measurement

The student will:

*1. measure lengths (metric and English), temperatures, angles

and weight

2. find perimeters, areas and volumes without computing by comparing with standard units

*3. convert measurements on a map or scale drawing to actual distances

*4. convert commonly used units of measurement within a measurement system, e.g., g to kg, ft. to in.

*5. determine time elapsed

D. Probability and Statistics

The student will:

*1. determine the probability of equally likely events, e.g., tossing a coin or a die

2. compute the mean, median, mode and range for given data

3. given a set of whole numbers and/or decimals, find the average

*4. read and interpret graphs, such as histogram, circle, bar and line graphs

5. construct an appropriate graph given a set of data

3. MATHEMATICAL UNDERSTANDING

Mathematical understanding refers to the explanation and interpretation of mathematical knowledge. Mathematical understanding relies primarily on translation processes. The mathematical knowledge can be expressed in words, symbols or figures; and the translation may be within or between any of these modes of expression. Mathematical understanding involves memory processes as well as processes of associating one item of knowledge with another.

Exercises that assess mathematical understanding require that a person provide an explanation or an illustration for one or more items of knowledge. They require the transformation of knowledge but not the application of that knowledge to the solution of a problem. An example of an exercise involving explanation would be one that asks why a certain graph is not the graph of a function. Examples of exercises involving interpretation would be ones that ask for a drawing of an array to represent six times seven or that ask for an equation to represent the information in a word problem.

The student will demonstrate a competency in the following areas:

A. Algebraic Expressions

1. translate a simple verbal expression into an algebraic expression
2. represent a mathematical sentence by listing ordered pairs
3. translate an equation into a verbal statement

B. Graphing

*1. plot given ordered pairs as points on a coordinate plane
2. give the ordered pair for any point on the coordinate plane
*3. plot points and intervals on a number line
4. represent a simple algebraic equation with an appropriate graph on a coordinate plane

C. Numeration

1. write the base ten numeral for a given base two numeral
*2. correctly match number sentences with subtraction arrows on a number line
*3. change an expanded form of a numeral to its standard form
*4. when given a drawing of a whole divided into equal parts, identify the fractional part which is shaded
*5. correctly match number sentences with set and subset removal drawings (subtraction)

D. Computers

*1. read a flow chart of common everyday occurrences
2. read flow charts of mathematical processes

4. MATHEMATICAL APPLICATION AND PROBLEM SOLVING

Mathematical application refers to the use of mathematical knowledge, skill and understanding. Mathematical application relies on memory, algorithmic, translation and judgment processes.

Exercises that assess mathematical application require a sequence of processes that relate to the formulation, solution and interpretation of problems. The processes may include recalling and recoding knowledge, selecting and carrying out algorithms, making and testing conjectures and evaluating arguments and results. An exercise might require the solution of a standard problem on proportion, or it might require the demonstration that two geometric figures are congruent. It might require the estimation of the amount of carpet needed for a room or the formulation of a problem given a graph of statistical data. Exercises assessing mathematical application may vary from routine textbook problems to exercises dealing with mathematical arguments. In these exercises the person is not told exactly what to do or how to do it. He or she must use reasoning and decision-making processes as well as mathematical knowledge, skill and understanding.

A. Recognizing General Patterns and Reasoning in Arithmetic Problems

The student will:

*1. find the answer to a verbal problem which involves quantitative relationships familiar to the student, e.g., money measurement

2. find irrelevant data included in a problem

3. identify problems in which there are insufficient data given and indicate what data are needed to solve the problem

4. find examples that verify a statement such as, "The sum of any two odd numbers is an even number."

5. find examples that contradict a statement such as, "All prime numbers are odd numbers."

*6. given a familiar problem, will recognize a best estimate of the results

7. recognize a pattern in numbers, then extend the pattern and/or generalize the sequence

8. recognize a pattern in geometric figures, then extend the pattern, e.g., the number of diagonals from one vertex in a polygon

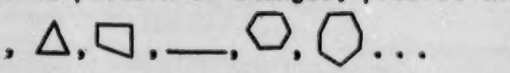
9. visualize a three-dimensional drawing and recognize other views of the same item

B. Geometric Generalizations and Relations

The student will:

1. make sketches of a geometric situation ("Draw a picture!") to provide a pattern from which a generalization can be stated

*2. collect data to find a generalization, e.g., "Measure and find the sum of the angles of a polygon."

3. given a geometric pattern of changes, predict the missing pattern, e.g., 

*4. given measurements, compute the perimeters and areas of triangles, squares, rectangles and circles. Give formulas for triangles and circles

*5. given measurements, compute the volumes of cubes and rectangular prisms (boxes)

6. given formulas and measurements, compute the:

*a. areas of trapezoids, parallelograms, regular polygons, etc.

b. volumes of cylinders, pyramids, cones, spheres, etc.

*7. given two intersection lines and the size of one of the angles formed, determine the sizes of the other angles in the figure

*8. given the perimeter of a square, find the length of one side

9. given several 30-60-90 degree triangles, determine the relationship between the shortest and longest sides

10. given several quadrilaterals and lines joining the midpoints of adjacent sides, determine the properties of the figure formed by the lines

C. Algebraic Problems

The student will:

1. identify the formula or equation, such as d = rt, which is involved in the relationship described in a problem

*2. solve a problem which is based on a relationship which can be expressed by an equation

3. discover the relationship or equation which will solve a problem by considering special cases involving small numbers

D. Consumer Buying and Money Problems

The student will:

*1. determine the cost per unit, e.g., cost of one gram (ounce) if 7.5 grams (ounces) costs 45¢

*2. determine the discount on sale items

*3. determine if a savings is made by buying "large economy size" rather than smaller size units

4. determine different ways to make totals such as 7¢, 13¢, 19¢, etc.

*5. given an amount to be paid, make change from a larger amount

*6. given an annual interest rate, find the interest paid on a given amount of money

E. Tables

The student will:

*1. read and interpret a table of data

2. given a set of data, construct a table

3. interpolate to find a value that is not included in a table

*4. determine sales tax by using tax tables

F. Scale Drawings and Maps and Planning a Trip

The student will:

1. locate points on a highway map with ordered pairs
2. find the solution to a problem situation by making a scale drawing and measuring the required length, e.g., find the unknown height of a pole
3. given certain measurements, determine unknown distances by scale drawing on a graph paper
4. given certain measurements, directions and locations of objects in a given area, choose a map (given 4 choices)
5. given a scale drawing, determine distances, directions and locations
6. determine the cost per kilometre (mile) for automobile travel considering gasoline, oil repairs, depreciation, insurance and taxes
7. read travel schedules for plane, train and bus
8. determine distances on a highway map for driving

G. Problem Solving

*The student will solve problems requiring analysis, but at a level of comprehension appropriate for eighth grade students, e.g., determine the number of persons in an auditorium with 24 rows and 12 seats in a row if 37 seats are empty

H. Novel Problems and Puzzles

The student will solve problems such as the following:

1. given 4 squares, how many polyominoes can be formed
2. which of these networks can be traced without retracing a line and without lifting the pencil off the paper
3. how many coins can be placed on these grids so there is only one coin on each row, each column and each diagonal
4. find the missing numbers in a magic square

5. APPRECIATION AND USE OF MATHEMATICS

A. Recognizing the Importance and Relevance of Mathematics to the Individual and to Society

From age 9 and up, there should be a recognition of the importance and relevance of mathematics to the individual and to society. This subobjective does not necessarily involve enjoyment of mathematics or participation in the development of ideas, but rather it focuses on the acceptance of mathematics as being worthwhile--i.e., the individual recognizes that mathematics is necessary whether or not he/she uses it or enjoys studying it. For example, the individual should recognize the contribution that mathematics has made to the progress of civilization, especially the sciences. There should also be the appreciation of the elegance, economy and techniques of mathematics. Of course, the level of sophistication of such appreciations should increase with age; nevertheless, some manifestation of these attitudes should appear at all age levels.

B. Enjoyment of Mathematics

In addition to having an appreciation for the importance of mathematics, the individual should also enjoy the subject and its specialized techniques, e.g., using compasses and working with numbers. Emphasis should be placed on the enjoyment involved in acquiring a knowledge of mathematics and in the satisfaction gained from using it rather than on the amount that is learned. The corollary of this is also important--i.e., the individual should not hate nor fear mathematics. These attitudinal goals are especially important during the school years since they are likely to influence how much mathematics an individual is willing to study and, therefore, have at his/her disposal.

C. Using the Content and Techniques of Mathematics

When the mathematics is relevant and appropriate, individuals should use what they have learned. Because the amount of knowledge varies with age level, evidence of a willingness to use mathematics will take different forms at different age levels.

D. Participation in the Learning of Mathematics Beyond That Which is Merely Required and Actively Seeking to Further Personal Development in the Area of Mathematics

The fourth subgoal related to the individual's development of a curiosity about mathematics as well as a readiness to engage in activites in this area, i.e., independent of school and/or job assignments. In contrast to the objectives in other categories, independent action rather than reaction is stressed. This goal emphasizes that the individual should actively seek participation and further development of his/her skills in mathematics (as indicated by such things as reading about the "new" math and tackling strange looking problems). This is opposed to merely passing judgment or using the principles learned when this was required. It is expected that such interests will not develop before the age of 13; however, once developed, they will probably carry into adult life.

CHAPTER III

PERFORMANCE BY OBJECTIVES

This portion of the report summarizes results by objective. At each grade level, exercises were selected to measure specific objectives at each grade level. The following four basic level of competencies were tested.

1. MATHEMATICAL KNOWLEDGE

   Assessment of this ability refers to the recall and recognition of mathematical ideas expressed in words, symbols or figures. Mathematical knowledge relies, for the most part, on memory processes. It does not ordinarily require other more complex mental processes.

2. MATHEMATICAL SKILL

   Mathematical skill refers to the routine manipulation of mathematical ideas. Mathematical skill relies on algorithmic processes. An algorithm is a standard procedure that always leads to an answer. Mathematical skill requires the recollection of how to use the algorithm.

3. MATHEMATICAL UNDERSTANDING

   This third level includes exercises that require a person to provide an explanation or an illustration for one or more items of knowledge. They require the transformation of knowledge but not the application of that knowledge to the solution

of a problem.

4. MATHEMATICAL APPLICATION AND PROBLEM SOLVING

Exercises that assess this ability require a sequence of processes that relate to the formulation, solution and interpretation of problems. The process may include recalling and recoding knowledge, selecting and carrying out algorithms, making and testing conjectures and evaluating arguments and results.

When reviewing results, several factors should be kept in mind:

-The sampling procedures permit reporting results that reliably represent performance of all North Dakota fourth and eighth graders.

-These results can be used to identify success and problem areas in achievement of students on a statewide basis.

-While performance results on a school basis will be provided to participating schools, great care should be exercised in making judgements about performance of any single school. In some instances, the sample may be only three pupils and a small number of incorrect answers translate into a large reduction in success percentages.

-Findings reported are based on a single assessment and do not purport to show longitudinal comparisons.

-Different tests were administered at the fourth and eighth grades. The intent was to measure performance at both a basic level and also at a level of difficulty at the appropriate level for the two grades. Several of the items were the same for both grades for comparison purposes. However, such comparisons are not

intended to indicate statistically significant differences.

-The National Assessment items were also included for comparison purposes, but differences were not calculated on a statistical basis.

-A large number of the objectives were assessed using only one item.

-The nature of each objective should be taken into account in determining whether a high priority need exists in the state or in a particular school system.

The results by objective for grade 4 are presented in Table 1. The results for grade 8 are presented in Table 2.

TABLE 1
Summary of Fourth Grade Student Performance by Objective

	Objective	No. of Items	State Success Level	Number of Students
The student will:				
1A1	Recall the basic facts of addition	1	95.7	282
1A2	Recall the basic facts of subtraction	1	95.3	365
1A3	Recall the basic facts of multiplication	1	95.9	343
1A4	Recall the basic facts of division	2	85.4	673
1B1	Identify by written word, the counting numbers to 100	1	96.4	365
1B2	Identify by numerical symbol, the counting numbers to 1,000	3	95.7	933
1B5	Recognize examples of even and odd numbers	2	94.1	708
1B7	Identify the position of an item in a set of objects (ordinality)	2	95.2	590

Table 1 (cont.)

	Objective	No. of Items	State Success Level	Number of Students
1C1	Recognize examples of geometric shapes	3	91.6	933
1C2	Identify geometric concepts: point, line, curve, cube	1	94.2	365
1D3	Identify a coin and give its value	2	95.8	708
1D4	Recognize appropriate units of measure	3	90.3	590
1D5	Convert units of measurement within a measurement system	1	51.0	365
1E1	Recognize proper use of symbols: ( ), =, > and <	2	74.3	708
2A1a	Add two whole numbers, with or without regrouping	3	[illegible].2	990
2A1b	Add three whole numbers, with or without regrouping	1	90.9	308
2A1c	Add decimals	3	73.7	933
2A2	Find the difference between two numbers less than 1,000	4	87.0	1,298
2A3	Name the product of a two digit number and a one digit number	4	91.4	1,298
2A4	Given a dividend less than 50 and a one digit divisor, name the quotient and remainder	2	97.5	590
2A5	Round off to the nearest hundred and the nearest ten	1	63.6	343
2A9	Count by 2's and 3's	2	91.4	708
2C1	Measure a given line segment to the nearest unit	1	95.5	308
2C3	Be able to tell time	2	92.0	590

Table 1 (cont.)

	Objective	No. of Items	State Success Level	Number of Students
2C4	Correctly read the temperature on a thermometer	2	91.2	708
2C5	Indicate the value of coins in cents	2	91.0	590
2D1	Be able to recognize data on a given graph	5	69.5	990
3A2	Change a verbal statement to a mathematical sentence	4	81.2	1,298
3A5	Give the sum of any number and zero as the given number	1	93.3	282
3A8	Give the product of any number and 1 as the given number	1	98.4	365
3A9	Recognize where to use <, = or >	1	86.5	282
3C1	Assign the correct place value to digits in a numeral	3	82.2	1,016
3C2	Rewrite a number in expanded notation	1	82.8	343
3C3	Correctly match number sentences with set combination drawings (addition)	1	96.8	308
3C4	Correctly match number sentences with set and subset removal drawings (subtraction)	1	85.4	343
3C6	Indicate that repeated addition can be done by multiplication	1	89.6	308
3C10	Find the fractional part of a number	1	76.2	282
3C12	Identify the fractional part of a drawing which is shaded	2	74.8	673
3C15	Correctly match number sentences with subtraction arrows on a number line	1	17.7	282
3D1	Select the appropriate metric unit to use when measuring length or distance	2	73.3	590

Table 1 (cont.)

	Objective	No. of Items	State Success Level	Number of Students
3D2	Select the appropriate metric unit to indicate mass	1	43.8	365
3D3	Determine time elapsed	1	72.3	343
4A1	when given a problem, estimate the answer	1	89.3	308
4A2	Arrange counting numbers from smallest to largest	1	92.4	343
4A3	When given a simple word problem, find the solution	5	59.5	1,298
4A4	Determine the distance traveled from two odometer readings	1	73.4	308
4A5	Correctly identify extraneous data	1	57.0	365
4A6	Identify data needed to solve a problem	1	83.4	343
4A7	Arrange fractions from the smallest to the largest (identify the smallest)	1	61.1	365
4B1	Find the perimeters of rectangles, triangles and other polygons	3	59.5	990
4C1	Find the total cost of several items	1	95.5	308
4C2	Make change from a dollar for a purchase less than a dollar	2	76.5	647
4C3	Make change from an amount less than a dollar	1	93.5	308

TABLE 2
Summary of Eighth Grade Student Performance by Objective

	Objective	No. of Items	State Success Level	Number of Students
The student will:				
1A	Recall the basic facts of addition, subtraction, multiplication and division	1	94.3	299
1B2	Recognize examples of terms	1	82.2	409
1B4	Identify the value of the digits of a number written in base 10	3	83.1	1,031
1B7	Read and write Roman numerals	1	88.7	372
1C1	Recognize examples of geometric terms	1	92.2	372
1C3	When given possible difinition of geometric terms, select the correct term	1	76.3	409
1D1	Identify metric units in terms of length, mass and capacity	3	82.7	1,068
1D2	Identify values associated with the metric prefixes	1	33.1	299
1D3	Identify the meaning of area and volume	1	55.4	372
1D4	Identify the quantitative relationships among units of measurement	2	97.3	708
1E1	Recognize and correctly use symbols =, > and <	1	84.7	372
2A1a	Demonstrate computation skills in addition and subtraction of whole numbers	8	96.0	1,440
2A1b	Demonstrate computation skills in addition and subtraction of decimals	4	94.3	1,440

Table 2 (cont.)

	Objective	No. of Items	State Success Level	Number of Students
2A1c	Demonstrate computation skills in addition and subtraction of fractions and mixed numbers	3	90.5	1,031
2A1d	Demonstrate computation skills in the addition and subtraction of integers	2	59.7	781
2A2a	Demonstrate computation skills in the multiplication of whole numbers	3	96.9	1,031
2A2b	Demonstrate computation skills in the multiplication of decimals	2	89.4	769
2A2c	Demonstrate computation skills in multiplying fractions and mixed numbers	2	81.7	671
2A2d	Demonstrate computation skills by multiplying integers	2	76.4	671
2A3a	Demonstrate computation skills in the division of whole numbers	2	88.4	769
2A3b	Demonstrate computation skills by dividing decimals	3	84.6	1,031
2A5	Change a decimal to an equivalent percent and vice versa	4	84.7	1,068
2A7	Arrange decimals and whole numbers according to numerical order	1	90.3	372
2A9	Round off numerals to nearest ten, hundred, thousand or million	2	85.9	769
2A10	Round off decimal numerals to nearest tenth, hundredth or thousandth	1	78.2	409
2A13	Determine G.C.F., L.C.M. of whole numbers with 2 or less digits	2	80.3	769

Table 2 (cont.)

	Objective	No. of Items	State Success Level	Number of Students
2A14	Solve percentage problems	4	54.1	1,440
2B3	Find rational number solutions for simple linear equations	1	83.9	372
2B5	Given three values in a proportion, solve for the fourth	2	55.9	769
2C1	Measure lengths, temperatures, angles and weight	4	94.2	1,068
2C3	Convert measurements on a map scale drawing to actual distances	2	77.5	769
2C4	Convert commonly used units of measurement within a measurement system	2	64.1	372
2C5	Determine elapsed time	4	49.4	1,068
2D1	Determine the probability of equally likely events	2	64.1	671
2D4	Read and interpret graphs	3	90.5	1,068
3B1	Plot given ordered pairs as points on a coordinate plane	1	48.6	360
3B3	Plot points and intervals on a number line	2	91.5	708
3C2	Correctly match number sentences with subtraction arrows on a number line	1	56.2	409
3C3	Change an expanded form of a numeral to its standard form	1	91.1	372
3C4	When given a drawing of a whole divided into equal parts, identify the fractional part that is shaded	1	91.4	372
3C5	Correctly match number sentences with set and subset removal drawing (subtraction)	1	92.5	360
3D1	Read a flow chart of common everyday occurrences	1	67.7	372

Table 2 (cont.)

	Objective	No. of Items	State Success Level	Number of Students
4A1	Find the answer to verbal problems	2	96.4	372
4A6	Given a familiar problem, recognize a best estimate of the results	3	71.0	1,031
4B2	Collect data to find a generalization	1	55.5	409
4B4	Given measurements, compute perimeter and areas	6	66.7	1,141
4B5	Given measurements, compute the volumes of cubes and rectangular prisms	1	77.2	372
4B6a	Given formulas and measurements, compute areas	1	62.5	299
4B7	Given two intersecting lines and the size of one of the angles formed, determine the sizes of the other angles in the figure	1	84.7	360
4B8	Given the perimeter of a square, find the length of one side	1	81.7	372
4C2	Solve a problem which is based on a relationship which can be expressed by an equation	1	71.2	299
4D1	Determine the cost per unit	4	67.7	1,068
4D2	Determine the discount on sale items	2	49.2	708
4D3	Determine savings on volume buying	1	29.0	372
4D5	Given an amount to be paid, make change from a larger amount	1	95.8	409
4D6	Find interest paid	3	80.5	1,068
4E1	Read and interpret a table of data	1	73.6	299
4E4	Determine sales tax by using tax tables	1	82.9	409

Table 2 (cont.)

	Objective	No. of	State Success Level	Number of Students
4G	Solve problems requiring analysis	5	79.1	1,031

CHAPTER IV

PERFORMANCE ON OVERLAP ITEMS

Twenty-five of the items appearing on the fourth grade level were also assessed at the eighth grade level. An examination of student performance on all 25 items is as follows:

On the left side of the page each item is reproduced as it appeared in the student test booklets. At the right side of the page is a small box for each item indicating student performance for each grade level.

1. Which is longer?

   (A) 2 feet
   (B) 1 yard

Grade	Percent Correct
4	87.6
8	97.8

(4A-5) (8A-25)

2. Multiply:

$$\begin{array}{r} 408 \\ \times\ 4 \\ \hline \end{array}$$

   (A) 1632
   (B) 16,032
   (C) 1602
   (D) 1902

Grade	Percent Correct
4	87.2
8	97.7

(4A-9) (8D-1)

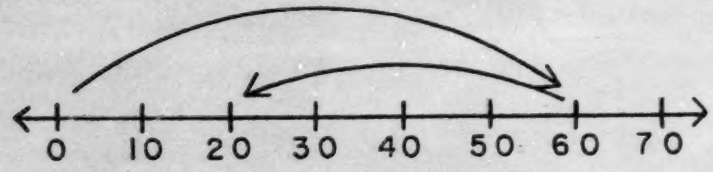
3. What is the sum of 21 and 54?

   (A) 85
   (B) 75
   (C) 84
   (D) 73

Grade	Percent Correct
4	91.8
8	97.1

(4A-14) (8A-10)

4. Look at the number line below:



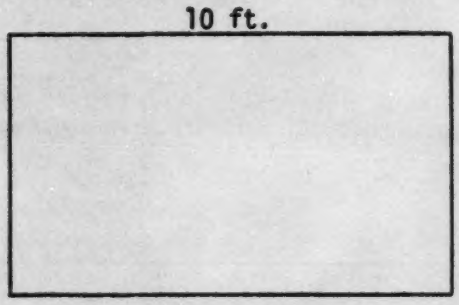
Grade	Percent Correct
4	17.7
8	56.1

(4A-18) (8A-28)

Which number sentence describes the number line operation?

(A) 60 + 20 = 80
(B) 80 - 60 = 20
(C) 60 - 40 = 20
(D) 60 - 20 = 40

5. What is the distance around this rectangle?



Grade	Percent Correct
4	72.9
8	91.4

(4B-3) (8B-28)

(A) 16 feet
(B) 30 feet
(C) 32 feet
(D) 36 feet

6. Subtract 237 from 504.

(A) 337
(B) 457
(C) 267
(D) 333

Grade	Percent Correct
4	80.5
8	98.9

(4B-6) (8B-19)

7. Which one of the following is the sum of three hundreds, eight tens and four ones?

(A) 15
(B) 384
(C) 300,804
(D) 3084

Grade	Percent Correct
4	82.5
8	91.1

(4B-8) (8B-11)

8. What does the symbol (<) mean?

(A) Equal to
(B) Less than
(C) Greater than

Grade	Percent Correct
4	61.4
8	84.7

(4B-9) (8B-21)

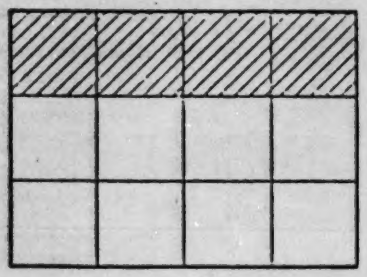
9. Paul has 21 stamps in his collection. He buys 54 more from a stamp dealer. How many does he have after he buys them?

(A) 33
(B) 75
(C) 73
(D) 85

Grade	Percent Correct
4	94.5
8	98.4

(4B-16) (8B-24)

10. What fractional part of this figure is shaded?



Grade	Percent Correct
4	53.4
8	91.4

(4B-20) (8B-18)

(A) $\frac{12}{4}$ or $\frac{3}{1}$

(B) $\frac{3}{4}$ or $\frac{9}{12}$

(C) $\frac{1}{3}$ or $\frac{4}{12}$

(D) $\frac{4}{6}$ or $\frac{8}{12}$

11. How many centimetres are 5 metres 34 centimetres?

(A) 39

(B) 84

(C) 534

(D) 50,034

Grade	Percent Correct
4	51.0
8	72.8

(4B-24) (8B-13)

12. Do the following subtraction:

$$\begin{array}{r} 1{,}054 \\ -\ 865 \\ \hline \end{array}$$

(A) 189

(B) 1299

(C) 289

(D) 179

Grade	Percent Correct
4	78.4
8	92.5

(4C-1) (8C-24)

13. In the number 23,486 what digit is in the thousands place?

(A) 2
(B) 3
(C) 4
(D) 8

Grade	Percent Correct
4	81.3
8	89.4

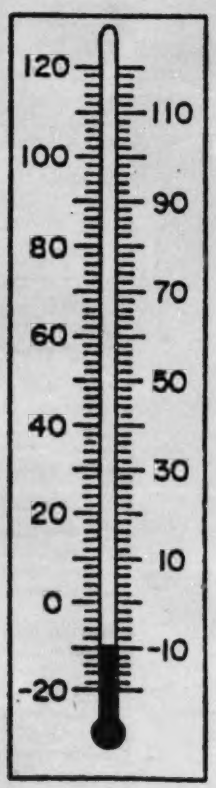
(4C-4) (8C-2)

14. What temperature is shown on this thermometer?

(A) -10^{0}
(B) -5^{0}
(C) 5^{0}
(D) 10^{0}

Grade	Percent Correct
4	84.8
8	97.5

(4C-9) (8C-27)



15. Add:

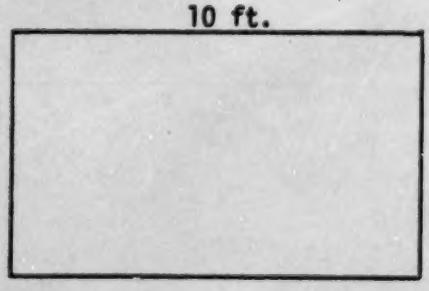
$$\begin{array}{r} 38 \\ +19 \\ \hline \end{array}$$

(A) 47
(B) 57
(C) 58
(D) 49

Grade	Percent Correct
4	94.5
8	96.8

(4C-14) (8C-26)

16. What is the PERIMETER of this rectangle?



(A) 16 feet
(B) 30 feet
(C) 32 feet
(D) 36 feet

Grade	Percent Correct
4	60.6
8	75.6

(4C-17) (8C-3)

17. Select a subtraction sentence telling about this picture.

(A) 4 - 3 = 1
(B) 7 - 4 = 3
(C) 7 - 3 = 4
(D) 3 - 2 = 1

Grade	Percent Correct
4	85.4
8	92.4

(4C-19) (8C-15)

18. In the number 2079 what digit is in the tens place?

(A) 2
(B) 0
(C) 7
(D) 9

Grade	Percent Correct
4	82.8
8	88.7

(4D-1) (8B-27)

19. Which unit would you use to measure the length of your thumb?

(A) Centimetre
(B) Metre
(C) Kilometre
(D) Millimetre

Grade	Percent Correct
4	78.9
8	76.6

(4D-6) (8D-29)

20. Subtract:

$$\begin{array}{r} 36 \\ -19 \\ \hline \end{array}$$

(A) 27
(B) 45
(C) 17
(D) 55

Grade	Percent Correct
4	94.2
8	94.3

(4D-11) (8D-30)

21. Sue had a hot dog, french fries and milk. How much did she spend?

MENU

Hamburger	.85	Milk	.20
Hot Dog	.70	Soft Drink	.15
Grilled Cheese Sandwich	.55	Milk Shake	.45
French Fries	.40	Ice Cream	.40

(A) $1.20
(B) $1.30
(C) $1.40
(D) $1.50

Grade	Percent Correct
4	95.0
8	97.3

(4D-12) (8D-16)

22. Which holds more water?

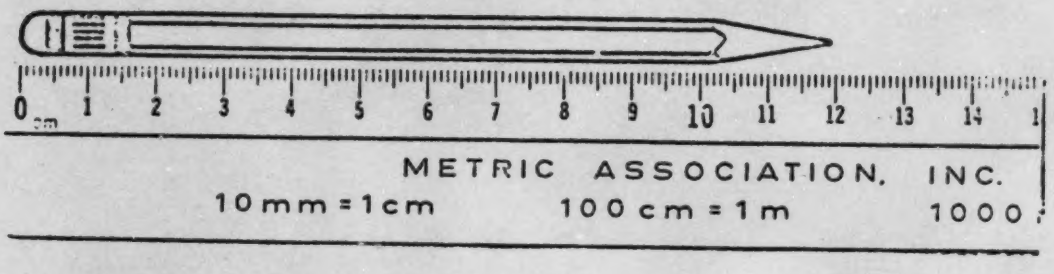
(A) 3 pints

(X) 2 quarts

Grade	Percent Correct
4	91.9
8	96.7

(4D-13) (8D-11)

23. What is the length of this pencil to the nearest centimetre?



(X) 12 centimetres

(B) 11 centimetres

(C) 10 centimetres

(D) 13 centimetres

Grade	Percent Correct
4	95.5
8	94.1

(4D-23) (8A-9)

24. If you spent 72 cents, how much change should you get back from one dollar?

(A) 1 quarter and 2 pennies

(X) 1 quarter and 3 pennies

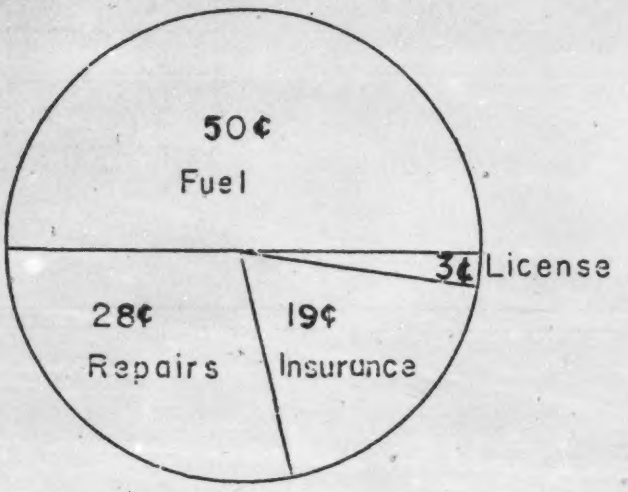
(C) 1 dime and 2 nickels

(D) 1 quarter and 1 nickel

Grade	Percent Correct
4	69.9
8	95.8

(4A-20) (8A-7)

25. How many cents of each dollar of operating costs went for insurance?



John's expenses for operating his minibike.

(A) 50¢
(B) 28¢
(C) 19¢
(D) 3¢

Grade	Percent Correct
4	71.7
8	87.2

(4C-25) (8C-12)

CHAPTER V

NORTH DAKOTA VERSUS U.S. PERFORMANCE COMPARISON

Several of the mathematics items completed by North Dakota students were identical in form to those used by National Assessment of Educational Progress (NAEP). The use of these items facilitates the comparison of the mathematics performance of North Dakota students at grade four and eight with the performance of similar students in the nation. The test items are identified according to the objective they measure and their location in the specific test booklet. The comparisons of North Dakota student performance on objectives and items are presented in Table 3.

TABLE 3
Comparison of Fourth Grade Student Performance on Objectives with National Performance

Objective		Form-Item No.	Performance Level In Percents	
			NAEP	N.D.
The student will:				
1A1	Recall the basic facts of addition	4A-13	95.6	95.7
1A2	Recall the basic facts of subtraction	4B-21	88.6	95.3
1A3	Recall the basic facts of multiplication	4C-20	82.6	95.9
1B2	Identify by numerical symbol the counting numbers to 1,000	4C-13	87.8	95.7

Table 3 (Cont.)

	Objective	Form-Item No.	Performance Level In Percents	
			NAEP	N.D.
1C1	Recognize examples of geometric shapes	4A-22	48.4	76.2
1C2	Identify geometric concepts: point, line, curve, cube	4B-22	97.9	94.2
1D3	Identify a coin and give its value	4B-5	88.8	96.7
1D4	Recognize appropriate units of measure	4A-5 4D-13	81.8 85.2	87.6 91.9
1E1	Recognize proper use of symbols: ( ), =, > and <	4B-9 4C-21	45.1 75.7	61.4 74.3
2A1a	Add two whole numbers, with or without regrouping	4A-14 4B-1	79.9 90.6	91.8 95.3
2A1c	Add decimals	4A-7 4C-22 4D-10	43.3 18.2 13.5	82.6 73.7 47.4
2A2	Find the difference between two numbers less than 1,000	4B-6 4C-1	33.9 27.2	80.5 87.0
2A3	Name the product of any two digit number and a one digit number	4A-9 4C-12 4D-2	88.5 89.2 25.2	87.2 91.4 96.1
2C4	Correctly read the temperature on an indicated thermometer	4C-9	80.8	91.2
2C5	Indicate the value of coins in cents	4A-11	74.3	89.4
2D1	Be able to recognize data on a given graph	4A-16 4B-15	60.6 32.2	85.5 58.9
3A2	Change a verbal statement to a mathematical sentence	4A-4	50.4	66.7
3A5	Give the sum of any number and zero as the given number	4A-1	94.3	93.0
3C1	Assign the correct place value to digits in a numeral	4B-8 4C-4 4D-1	73.8 57.6 62.8	82.5 82.2 82.8

Table 3 (Cont.)

	Objective	Form-Item No.	Performance Level In Percents	
			NAEP	N.D.
3C2	Rewrite a number in expanded notation	4C-18	74.3	82.8
3C6	Indicate that repeated addition can be done by multiplication	4D-15	73.3	89.6
3C10	Find the fractional part of a number	4A-23	25.5	76.2
3C12	Identify the fractional part of a drawing which is shaded	4B-20	22.7	53.4
3D1	Select the appropriate metric unit to use when measuring length or distance	4D-6	68.1	78.9
3D3	Determine time elapsed	4C-24	33.2	72.3
4A3	When given a simple word problem, find the solution	4A-15	45.9	79.4
		4B-16	85.4	94.5
		4C-3	19.4	59.5
		4D-17	3.0	30.8
4B1	Find the perimeters of rectangles, triangles and other polygons	4B-3	46.2	72.9
		4C-17	9.1	59.5
4C1	Find the total cost of several items	4D-12	62.8	95.5

TABLE 4
Comparison of Eighth Grade Student Performance on Objectives with National Performance

	Objective	Form-Item No.	Performance Level In Percents	
			NAEP	N.D.
1B4	Identify the value of the digits of a number written in base 10	8B-27 8C-2	84.5 86.6	88.7 89.4
1D1	Identify metric units in terms of length, mass and capacity	8D-29	84.5	76.6
1D4	Identify the quantitative relationship among units of measurement	8A-25 8D-11	93.3 93.5	97.8 96.7
1E1	Recognize and correctly use symbols =, > and <	8B-21	71.3	84.7
2A1a	Demonstrate computation skills in addition and subtraction of whole numbers	8A-10 8B-19 8C-24 8C-30	86.8 77.0 27.2 88.0	97.1 97.3 92.5 96.4
2A1b	Demonstrate computation skills in addition and subtraction of decimals	8A-19 8B-17	61.7 65.4	88.5 98.4
2A1c	Demonstrate computation skills in addition and subtraction of fractions and mixed numbers	8B-1 8C-22 8D-26	45.8 55.7 70.0	83.1 93.9 94.6
2A1d	Demonstrate computation skills in the addition and subtraction of integers	8A-30	25.7	41.1
2A2c	Demonstrate computation skills in multiplying fractions and mixed numbers	8B-26 8D-9	37.2 72.2	70.4 93.0
2A2d	Demonstrate computation skills by multiplying integers	8B-30	56.6	83.9
2A3a	Demonstrate computation skills in the division of whole numbers	8A-24 8C-28	51.5 73.5	85.3 91.4
2A7	Arrange decimals and whole numbers according to numerical order	8B-3	84.3	90.3

Table 4 (Cont.)

	Objective	Form-Item No.	Performance Level In Percents	
			NAEP	N.D.
2A14	Solve percentage problems	8A-1	5.2	65.8
		8B-5	42.2	67.2
		8C-21	10.5	56.4
		8D-4	7.7	27.4
2B3	Find rational number solutions for simple linear equations	8B-8	84.6	83.9
2C1	Measure lengths, temperatures, angles and weight	8C-27	91.0	97.5
2C4	Convert commonly used units of measurement within a measurement system	8B-13	42.4	72.8
2D1	Determine the probability of equally likely events	8D-23	11.2	43.5
3B3	Plot points and intervals on a number line	8A-15	85.1	87.8
		8D-22	55.0	95.3
3C3	Change an expanded form of a numeral to its standard form	8B-11	91.3	91.1
3C4	When given a drawing of a whole divided into equal parts, identify the fractional part that is shaded	8B-18	86.8	91.4
4A6	Given a familiar problem, recognize a best estimate of the results	8B-15	20.2	72.3
4B4	Given measurements, compute perimeter and areas	8B-6	34.6	80.4
		8B-28	73.2	91.4
		8C-3	52.9	75.6
4B5	Given measurements, compute the volumes of cubes and rectangular prisms	8B-16	62.3	77.2
4D2	Determine the discount on sale items	8D-31	16.4	37.8
4D6	Find interest paid	8D-16	81.1	97.3

Table 4 (Cont.)

	Objective	Form-Item No.	Performance Level In Percents	
			NAEP	N.D.
4G	Solve problems requiring analysis	8B-4	34.3	83.3
		8C-10	76.8	75.8
		8C-23	79.4	85.8
		8D-17	59.7	87.6

CHAPTER VI

ANALYSIS OF ERRORS ON MATHEMATICS EXERCISES

This part of the report contains information about student responses to mathematics assessment exercises. It is an attempt to call attention to the types of errors that were unusual or occurred most frequently.

Computation with Whole Numbers

Both fourth and eighth graders performed with strength on items related to basic knowledge, recall and computation with whole numbers. They performed less well on objectives judged to call for higher order cognitive skills applied to problem solving. The fourth grade students attained an achievement level of 90% on 50% of the objectives assessed while the eighth grade students attained this level on only 27% of the eighth grade objectives. It must be noted that these are not longitudinal data. Eighth grade students were tested on more difficult and diverse items than fourth grade students. The exception being the overlap items which were identical in the tests for both grade levels. On overlap items the percent of eighth graders getting them correct was always higher. An examination of student performance on all overlap items appears in Chapter IV.

Exercises involving addition of whole numbers did not appear to present difficulties to either fourth or eighth grade students. This was true for test items that required regrouping as well as for those that did not.

In response to the question

"What is the sum of 21 and 54?"

fourth grade students achieved at the 91.8% level while eighth grade students achieved at the 97.1% level. When the exercise was presented in the vertical format, fourth grade students achieved at the 95.3% level. While the difference in achievement levels between the horizontal and vertical format is small, the better performance suggests that exercises are more often presented in the vertical format.

Results on subtraction exercises show that most students are able to perform this operation. The largest number of incorrect answers involved mistakes in regrouping. For example, in responding to the following exercise

$$\begin{array}{r} 1054 \\ -\ 865 \\ \hline \end{array}$$

10 percent of the fourth grade students answered 289 and 8% thought the answer should be 1299. The correct answer for this exercise was provided by 78% of the fourth grade students and by 93% of the eighth grade students. Performance on subtraction exercises presented in the horizontal format followed a similar pattern to that of addition in the fourth grade. However, 80% of the fourth grade students were able to correctly subtract 237 from 504 in the horizontal format.

Fourth grade students had some difficulty with the multiplication exercise

$$\begin{array}{r} 408 \\ \times\ \ 4 \\ \hline \end{array}$$

which included a zero in the multiplicand. This difficulty did not persist through the eighth grade. While the zero created some difficulty when appearing in the middle of a three digit multiplicand, it presented no problem when appearing at the end of a two digit multiplicand. Ninety-six

percent of the fourth grade students were able to multiply 40 x 5 correctly. The change from vertical to horizontal format did not appear to create a problem when doing multiplication exercises.

Ninety-seven percent of the fourth grade students were able to do the exercise

$$3\overline{)15}$$

but only 74% were able to provide the correct answer to the exercise

$$3\overline{)47}$$

This may be due because exercises presented in the fourth grade often do not require the use of a remainder.

Ten percent of the eighth grade students gave 19 as the quotient for the exercise

$$38\overline{)3052}$$

instead of 109 which was given by 85% of the students. As in multiplication, the zero produces difficulty in computation.

Despite the errors mentioned, the results of the computation portion of the mathematics assessment indicate that, in general, students are quite proficient in computing with whole numbers. Improvements might be accomplished through more emphasis on the concept of regrouping and the use of the zero.

Numeration

Identifying counting numbers by written word or numerical symbol to the magnitude of 1,000 does not appear to be difficult for fourth and eighth grade students in North Dakota, however, some students were not as confident when identifying place values.

When given the exercise

"In the number 2079, what digit is in the tens place?"

82.8% of the fourth grade students selected 7 as the correct digit. This improved to 88.7% for eighth grade students.

When asked

> "Which one of the following is the sum of three hundreds, eight tens and four ones?"
>
> (A) 15
> (B) 384
> (C) 300,804
> (D) 3084

82.5% of the fourth grade students selected 384 which is correct, but 9% selected 300,804, a choice that does not indicate an understanding of place value. On the same problem 91.1% of the eighth grade students responded correctly.

In one exercise eighth grade students were asked

> "What digit is in the hundreths place in the number in the box?"

3.456

Of the eighth grade students, 21.7% were unable to distinguish between the hundreds and hundreths which was evidenced by the selection of the digit 4 as the correct answer.

The above results, along with the regrouping problems in the computation portion, indicate that some students have difficulty understanding place value. This situation might be improved if the concept were reviewed each year in North Dakota.

Roman numerals representing large numbers are no longer in common usage. However, Roman numerals representing smaller numbers are still used to identify chapters in books. When asked to identify IX, 88.7% of the eighth grade students were able to do so.

Computation with Decimals

Since the concept of decimals is usually developed in the upper grades, only addition exercises were included in the fourth grade assessment. Fourth grade pupils did well on decimal addition problems that did not produce more digits in the answer than were used for the numbers in the addends. But in the following exercise

$$.70 + .40 + .20 =$$

only 47.4% of the students selected the correct answer; 32.5% selected .130 and 19.2% selected 130. The decimal appearing at the beginning of each addend seems to have incorrectly suggested to the students that this should also be true for the sum.

Eighth grade students performed proficiently on addition and subtraction exercises in the vertical format with decimals aligned correctly, but the exercise

$$.35 + 1.4 =$$

10.3% of the eighth grade students gave the answer 4.9 which indicates some difficulty with decimal alignment--again, this appears to be related to the misunderstanding of place value.

The difficulty with placement of the decimal was also evident in multiplication and division exercises.

When asked to perform the following

$$\begin{array}{r} 5.3 \\ \times\ .9 \\ \hline \end{array}$$

12.5% of the eighth grade students selected 47.7 as the answer. This suggests that the students are following an error pattern similar to that found in addition and subtraction. However, 86.6% of the students selected the correct answer.

In response to the exercise

$.25\overline{)17.5}$

15.6% of the eighth grade students incorrectly selected .7 as the answer. When asked to solve the exercise

$.07\overline{).497}$

84.3% of the eighth grade students selected the correct answer; 8% selected .71 and 6% selected .071. The answers selected by students for the last two exercises suggests that there is some confusion regarding division of decimals.

Of the eighth grade students, 89.5% were able to change .45 to a percent, but this decreased to 81.1% when asked to change .0023 to a percent. However, when asked to change a percent to a decimal, 94.6% had no difficulty changing 50% to a decimal, but only 73.6% were able to change 235% to a decimal.

The results of the items discussed so far relating to computation with decimals would indicate that, except for the minority who are still confused on place value, this would not be considered to be an area of weakness. Improvement is, however, needed in solving percentage problems.

For example, in responding to the exercise below

"12 is 15% of what number?"

33.9% of the eighth grade students answered 70, 85 or 90. The range of responses would indicate some confusion exists on the proper approach to solving this type of problem.

This was not the case for the following exercise

"30 is what percent of 60?"

Twenty percent and the correct answer, 50%, were the responses most often selected. Twenty-four percent thought that the correct answer should be 20%. When asked

"What is 4% of 75?"

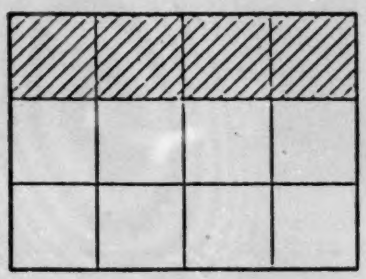
56.4% of the students provided the correct answer, but only 27.4% of the eighth grade students selected the correct answer to

"6 is what percent of 120?"

Concepts and Computation with Fractions

The meaning of "fraction" was investigated by presenting exercises as follows

"What fractional part of this figure is shaded?"



(A) $\frac{12}{4}$ or $\frac{3}{1}$

(B) $\frac{3}{4}$ or $\frac{9}{12}$

(C) $\frac{1}{3}$ or $\frac{4}{12}$

(D) $\frac{4}{6}$ or $\frac{8}{12}$

Of the fourth grade students, 53.4% correctly selected 1/3 or 4/12 while 30.7% selected 12/4 or 3/1. This was matched by a 91.4% correct response from eighth grade students.

Fourth grade students had no problem identifying

"Which has 1/2 shaded?"

2.3% (A)

96.1% (X)

1.0% (B)

.6% (D)

It would appear that eighth grade students understand the meaning of "fraction," and fourth grade students have no doubt been taught the basic idea of what a fraction is, but have not attained full understanding.

Exercises on operations with fractions were given mainly to eighth grade students. However, 76.2% of the fourth grade students assessed responded correctly to the exercise

"What is 1/4 of 8?"

The addition and subtraction of fractions and mixed numbers presented minimal difficulty for eighth grade students. More difficulty was encountered with the multiplication of fractions, especially problems containing mixed numbers.

When asked to solve the multiplication problem

$$\frac{1}{6} \times 7\frac{1}{3} =$$

(X) $1\frac{2}{9}$

(B) $\frac{7}{18}$

(C) $1\frac{7}{18}$

(D) $2\frac{5}{9}$

70.4% of the eighth grade students were able to give the correct answer. However, 7.8% of this group selected 7/18 as the answer, and 14% gave 1 7/18 as the correct response. The 21.8% who selected the incorrect responses were no doubt influenced by the fact that the denominator contained the number 18, the product of 6 x 3.

In a related exercise eighth grade students were asked to

"Find the lowest common denominator of 1/3 and 1/6."

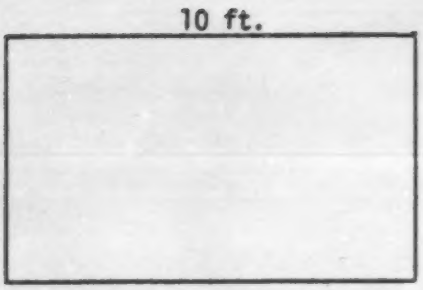
12% of the eighth grade students selected 12 as the correct answer. When asked to find the lowest common denominator of 1/6 and 1/8, 9% of the students gave an answer of 48 instead of the correct answer 24. In each case the students were aware that the number selected must be divisable by each of the denominators but did not select the lowest number possible.

A 1975 mathematics assessment of sixth grade students indicated concepts and computations with fractions as an area of need. However, considering the limited use of fractions in current mathematical processes, it would appear that North Dakota students are progressing in a satisfactory manner in this area.

Geometry

Students in each grade assessed were able to identify triangles, rectangles and circles, but 20.3% of the eighth grade students mistakenly identified the radius of a circle as its diameter. In a related concept, 76.2% of the fourth grade students were able to identify parallel lines.

In an effort to determine whether terminology effects results, fourth and eighth grade students were given the rectangle



and asked to find the "perimeter." The following results were obtained:

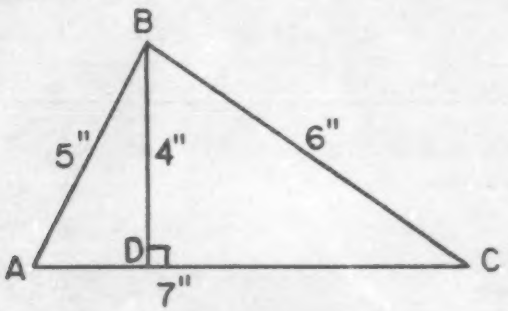
	4th Grade	8th Grade
(A) 16 ft.	35.3	17.2
(B) 30 ft.	1.5	2.8
(C) 32 ft.	60.6	75.6
(D) 36 ft.	2.3	4.2
N.R.	0.3	0.3

When asked to find "the distance around" the following resulted:

	4th Grade	8th Grade
(A) 16 ft.	24.4	4.0
(B) 30 ft.	1.1	1.9
(C) 32 ft.	72.9	91.4
(D) 36 ft.	1.6	2.4
N.R.		0.3

In another exercise

"What is the perimeter of the triangle ABC below?"



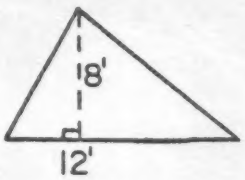
29.8% of the eighth grade students and 18.8% of the fourth grade students selected 22 inches as the correct answer. As in the exercises relating to the perimeter of the rectangle, students have a tendency to add all the numbers appearing in the problem. Either there is some confusion about the concept or students need to be more careful when reading the problem.

When finding the distance around was presented as the following "word" problem

> "Mr. Jones put a wire fence all the way around his rectangular garden. The fence is ten feet long and six feet wide. How many feet of fencing did he use?"

80.4% of the eighth grade students responded correctly. This can be compared to the 91.4% of the eighth grade students who responded correctly when asked to find the distance around the rectangle illustrated previously. It would appear that the visual display of the figure helped them to solve the problem.

Given the formula and measurements, eighth grade students were asked to find the area of the triangle



62.5% of the eighth grade students were able to do so while 22.1% multiplied 8 x 12 and neglected to divide by 2.

Algebra

While the first full year in elementary algebra is usually undertaken in grade 9, solving simple equations, selecting number sentences that fit the problem and other beginning ideas from algebra are taught in the elementary grades.

When asked which number sentence says the same thing as this statement

"If a number is multiplied by 6, the answer is 18."

81.3% of the fourth grade students selected the correct answer, but 10.2% selected $\square = 6 \times 18$. However, when fourth grade students were given the following exercise

"Jim had 100 marbles. He lost 84 of them. Which sentence shows how many marbles were left?"

96.7% of the fourth grade students selected $100 - 84 = \square$ as the correct answer.

In another exercise eighth grade students were asked for the value of x in the exercise

$$x - 3 = 7$$

83.9% were able to give the correct value of x; 14.8% gave the answer as 4. It appears that it is easier for students to use $\square$ instead of x to represent the unknown.

Finding the missing value (x) in a proportion may involve an algebraic process, but it helps to have an understanding of fractions.

When asked to find the missing value (x) of

$$\frac{12}{26} = \frac{18}{x}$$

only 36.7% of the eighth grade students gave 39 as the correct answer, and 43% incorrectly thougt it should be 43. Obviously, many of the students thought it should be an even number.

In another proportion exercise, one of the ratios was stated in lowest terms, and 75% of the students were able to provide the correct answer for the missing value.

Measurement

Exercises relating to measurement dealt with time, length, mass (weight), capacity and temperature. Exercises included both English and

metric units of measure.

Of the fourth grade students, 93.6% were able to tell time when the clock shows the hour, but only 84.5% did so when the hands on the clock indicated 15 minutes before the hour.

Students in both the fourth and eighth grade had difficulties with exercises which dealt with elapsed time, especially those that included more than one time zone.

In the first of a series of exercises when eighth grade students were asked to respond to the problem

> "A baseball game began at 2:30 p.m. and ended at 5:20 p.m. How many hours and minutes was the length of the game?"

70.9% of the eighth grade students were able to provide the correct answer. Since no error pattern was evident, it would appear that the remainder of the students were confused about how to attack the problem and guessed.

When the exercise included a change in time zones such as

> "The Central Time Zone is the next zone west of the Eastern Time Zone. A plane left a city at 11:45 a.m. Eastern Standard Time and reached its destination at 1:25 p.m. Central Standard Time. How long did the trip take?"

only 20.3% were able to provide the correct answer. Again, no error pattern was evident.

Finally, when asked

> "When it is 5:00 p.m. Central Standard Time, what time is it in the Mountain Standard Time Zone?"

58% of the eighth grade students answered correctly. The lack of knowledge about time zones provides some explanation for the low score on the previous exercise.

Sometimes the introduction of an additional foil to a multiple choice question produces interesting results. This is the case in the following exercise

"Which unit would you use to measure the length of your thumb?"

When fourth grade students were asked to select the correct answer from

(A) Centimetre
(B) Metre
(C) Kilometre

78.9% of the students selected the centimetre as the correct response. Eighth grade students were given the same exercise except that the additional foil, (D) Millimetre, was provided. Of the students 76.6% selected the centimetre as the correct response and 14.4% indicated that the millimetre would be the appropriate unit.

North Dakota fourth and eighth grade students quite readily identified the quantitative relationships among units of measure. They understand that two quarts hold more water than do three pints and that one yard is longer than two feet. They also know how to read a thermometer, measure the length of an object or read the weight indicated on a scale.

Students did not do as well when asked to convert units of measurement within a measurement system.

Of the eighth grade students, 72.8% were able to determine how many centimetres are in 5 metres 34 centimetres, but when asked

"Which one of the metric units below is equivalent to 1,000 millimetres?"

(A) 1 centimetre	3.2%
(B) 10 centimetres	25.0%
(C) 1 metre	55.4%
(D) 1 kilometre	16.1%

the number of successful students was reduced to 55.4%.

Metric measurement was identified as an area of strength in the 1978 assessment of eleventh grade mathematics. Results in the fourth and eighth grade assessment seem to be inconclusive, but with the increased emphasis

on metric studies, it is anticipated that student performance will improve during the next few years.

Problem Solving and "Word" Problems

A certain amount of resistance to solving "word" problems is common among students. Teachers have traditionally found the teaching of "word" problems a troublesome instructional task. Problem solving exercises were selected that would identify a student's ability to give estimations, to determine if the given data is sufficient to solve the problem and to select the appropriate problem solving procedures.

In the exercise

> "A book costs $5.25 and an airplane costs $2.75. How much more does the book cost than the airplane?"

79.4% of the fourth grade students subtracted correctly. However, 15.6% selected $3.50 which was the result of an error in regrouping.

When the following problem was given to eighth grade students

> "One rabbit eats 2 pounds of food each week. There are 52 weeks in a year. How much food will 5 rabbits eat in 1 week?"

87.6% of the students were able to ignore the extraneous information and provide the correct answer. Eight percent of the students selected 104 pounds which indicates that they thought something should be done with the 52 weeks.

On the following problem requiring more than one operation

> "Marie took four spelling tests. Each test had 30 words. On the four tests she spelled correctly the following number of words:
>
> 25, 23, 27, 24
>
> Altogether, how many did she miss on all four tests?"

55.4% of the fourth grade students correctly identified 21 as the correct answer.

When eighth grade students were given the following problem

> "A map of a state is to be drawn so that 1/4 inch represents five miles. If the real distance between two points in the state is 20 miles, how many inches apart should these two points be on the map?"

58.9% of the students correctly responded "1 inch," but 21.8% responded "5 inches," apparently multiplying 20 x 1/4.

Failure to read carefully was displayed by fourth grade students on the following exercise

> "If two apples cost 10¢, what will four apples cost?"

Only 37.3% of the students determined that each apple costs only 5¢ and the cost of four apples is 20¢. Of the students, 61.5% choose 40¢, the answer obtained by multiplying 4 x 10.

Consumer Buying and Money Problems

When fourth grade students were given the following problem

> "If you spend 72 cents, how much change should you get back from one dollar?"

83% of the students gave the correct answer. However, when they were asked to make change for the same problem in coins, the number of students giving the correct coins was reduced to 69.6%. Eighth grade students were able to identify the coins returned in change for the above exercise.

The ability of students to determine the best buy was examined with the following exercise

> "A large 16 ounce can of corn sells for 32 cents, a small 8 ounce can of corn sells for 24 cents. How much do you save per ounce by buying the large can of corn?"

Of the eighth grade students, 53.3% did not find the cost per ounce and incorrectly selected 8 cents, the difference between 32 and 24, as the answer.

Eighth grade students were asked to determine the discount on sale

items.

> "A sporting good store advertised '10% off on everything.' What is the sale price of a baseball glove that regularly sells for $25.00."

Of the students, 20.3% correctly identified the discount of $2.50, but the problem asked for the sale price of the glove which 60.6% of the students provided.

North Dakota students appear to know how to determine the cost of several items listed on a menu. They are also able to find the cost per unit. However, they seemed to have some difficulty when the problem required a comparison of unit costs.

This was again demonstrated in the following exercise

> "A housewife will pay the lowest price per ounce for rice if she buys it at the store which offers:"
>
> (A) 12 ounces for 40 cents
> (B) 14 ounces for 45 cents
> (C) 1 pound, 12 ounces for 85 cents
> (D) 2 pounds for 99 cents

Only 29% of the eighth grade students answered correctly. Since it was necessary to carry the amount two places, this problem alone may not be enough to determine student performance at this grade level.

When students were asked to determine the interest for a year on a $125 loan at a rate of 8 percent, 68.3% correctly identified the correct amount.

CHAPTER VII

SUMMARY AND RECOMMENDATIONS

When compared with national results, North Dakota children at both the fourth and the eighth grade levels performed very well overall on their respective assessment exercises. There are, however, several areas that need some attention. These areas were identified in the previous section and are discussed here with suggestions for improving overall achievement.

The computational difficulties that students had on the exam all seem to have a common link. That is the apparent lack of understanding of place value. This is common in children. Many indicate a superficial understanding of the concept of place value and when applying it to solve problems they make many errors. There also appears to be some problem with zero in computation. In looking at the type of errors made with zero, it would appear that this, too, is a place value problem. Students are not using the zeros to hold a place in numbers while they compute.

A possible solution to the place value problem would be to supply the children with more concrete activities using place value. The use of an abacus, counting frames, number base blocks or even pictures on graph paper should give the child a better feeling for this concept. The child who has solved problems requiring regrouping using an abacus tends to have a better feeling for this process when adding and can "borrow" more easily when the necessity arises in a subtraction problem.

The performance of students on geometry problems appears to indicate that a substantial number of them are unfamiliar with geometric terminology. While they know simple terms like triangle and circle, they find it easier to determine the "distance around" than finding the "perimeter" of a rectangle. The concepts tested on the geometry items were not difficult and both fourth and eighth graders should have had some exposure to them. This is not to say that students should spend hours doing geometric problems, but they should do some activities each year. Geometric activities in the lower grades can be fun and help students to like mathematics. Suggested activities here would include building geometric models, studying different shapes, comparing sides and area, diameter and circumference. Since there is not a great need to stress testing, students should be allowed to investigate, explore and develop knowledge.

At this time the metric system doesn't appear to have come as far as it should have in the elementary grades. This is unfortunate, as considerable progress has been made nationally in this area and North Dakota teachers have been provided with inservice opportunities. Children need to be able to work in the metric system. When they are adults, the metric system will be "the" system of measure in this country. Activities involving measuring distance, volume of liquids, weighing items and coverting within the metric system can be incorporated into any class's work. Children can be asked to read a thermometer every day and can make charts to record temperature. This constant re-exposure to the units in the system should raise student proficiency in this area.

Another apparent area of concern must be that of reading and interpreting graphs. With tables and graphs appearing in magazines, newspapers and on T.V., future citizens should all be able to understand them.

It would appear that many students do not get enough exposure to graphs. A recommendation here would be for all teachers (1-8) to find more ways to use graphs in class. As indicated previously, children can make and/or read graphs dealing with outside (or inside) temperature, make graphs comparing their heights, read graphs depicting populations or incomes in countries or states they study in social science and graph basketball standings or performance. They could even chart their own progress toward some goal (e.g., mastering mathematics facts or learning how to spell some words).

As indicated in the analysis of errors section, North Dakota children, like others nationally, are good at computing but are somewhat weaker in areas requiring higher levels of cognitive functioning. With the widespread use of calculators and computers, there will be more and more need for students to be able to set up problems. The machines will compute the answers. This means that students should be more able to solve word problems than in the past. The key here seems to be practice. Children learn how to do word problems by doing word problems. Our textbooks provide some word problems, but many times the additional problems at the end of the chapter or section are ignored. Students should be provided with an assortment of problems that they are capable of solving mathematically (easy computations) but must translate from words to mathematical equations before solving.

In summary, it would appear that a little more time spent on the topics mentioned here would only serve to improve the level of mathematics achievement in the state. It is easy to spend all the class time on computation and not include other topics. A well articulated school mathematics curriculum will assure that all topics at each level are given proper attention. It is easy at any level to "lose sight of the forest for the trees."